Fluid Dynamics: Physical ideas, the Navier-Stokes equations, and applications to lubrication flows and complex fluids

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Outline

• Part I: elementary ideas
  – A role for mechanical ideas
  – Brief picture tour: small to large lengths scales; fast and slow flows; gases and liquids

• Continuum hypothesis: material and transport properties
  Newtonian fluids (and a brief word about rheology)
  stress versus rate of strain; pressure and density variations;
  Reynolds number; Navier-Stokes eqns, additional body forces; interfacial tension: statics, interface deformation, gradients

• Part II: Prototypical flows: pressure and shear driven flows; instabilities; oscillatory flows

• Part III: Lubrication and thin film flows

• Part IV: Suspension flows - sedimentation, effective viscosities, an application to biological membranes
From atoms to atmospheres: mechanics in the physical sciences

• classical mechanics
  – particle and rigid body dynamics

• celestial mechanics
  – motion of stars, planets, comets, ...

• quantum mechanics
  – atoms and clusters of atoms

• statistical mechanics
  – properties of large numbers

Continuum mechanics:
(materials viewed as continua)

electrodynamics  solid mechanics
thermodynamics  fluid mechanics
A fluid dynamicist’s view of the world*

**

Galaxies

Mathematics

Astrophysics

Engineering

Geophysics

aeronautical

 Fluid dynamicist

biomedical

Chemistry

chemical

Physics

environmental

mechanical

Biology

Galaxies

Snow avalanche

* after theme of H.K. Moffatt

** http://zebu.uoregon.edu/messier.html

*** Courtesy of H. Huppert
Fluid motions occur in many forms around us:

Here is a short tour

(Water)

Big Waves

Little waves

Ship waves

Ref.: *An Album of Fluid Motion*, M. Van Dyke
Flow and design in sports

Cycles and cycling


Yacht design and the America’s Cup (importance of the keel)


*Bicycling* Feb. 1996
Swimming (large and small)

Micro-organisms: flagella, cilia

Four successive positions of the flagellum of a sea urchin sperm (Lytechinus pictus), captured by firing four flashes while the camera shutter was open.

Rowing

Basilisk or Jesus lizard

Speed vs. # of rowers?
T.A. McMahon, Science (1971)

Ref: McMahon & Bonner, On Size and Life; Alexander Exploring Biomechanics
Small fluid drops
(surface tension is important)

Water issuing from a millimeter-sized nozzle
(3 images on right: different oscillation frequencies given to liquid; ref: Van Dyke, An Album of Fluid Motion)

Bubble ink jet printer
(Olivetti)

also: deliver reagents to DNA (bio-chip) arrays

Three-dimensional printing -- MIT
(Prof. E. Sachs & colleagues)

Hagia Sophia (‘original’ in Istanbul Turkey)

5 inches
…. and a pretty picture …. 

A dolphin blowing a toroidal bubble
Elementary Ideas I

• A brief tour of basic elements leading through the governing partial differential equations

• Physical ideas, dimensionless parameters
2. CONTINUUM HYPOTHESIS:

(a) Variables such as pressure, density, temperature, velocity are continuous functions of position.

\[ p = \text{pressure} \]
\[ \rho = \text{density} \]
\[ T = \text{temperature} \]
\[ u = \text{velocity} \]

(b) cube 1\(\mu m\) on a side: averaging of large numbers
- \(3 \times 10^{10}\) water (\(\ell\)) molecules;
  \(10^{10}\) benzene molecules
- \(10^7\) gas molecules at STP
- \(10^3\) smaller for \(\ell \approx 0.1\mu m\) (\(10^3\) Å) on a side.

(c) local thermodynamics equilibrium assumed
\[ \Rightarrow \rho = \rho(p, T) \]
Elementary Ideas III

3. Molecular Dynamics

Coalescence of drops due to shear flow

Typical conclusion: calculations with $10^3$ molecules ($10$ per side of a cube!) agree with continuum descriptions.

THIN FILMS

Experiments on shearing between two molecularly smooth (mica) surfaces separated by thin films of organic liquids.

- Films > 10 molecular diameters can be described in terms of bulk properties.
- Thinner films: molecular ordering, quantization of some properties, "effective" viscosities > $10^5$ bulk value.
- Film with thickness less than 5 molecular diameters: "solid-like" response.

REFERENCE:
GEE, MCGUIGAN,
ISRAELACHVILI
& HOMOLA
J. CHEM. PHYS. (1990)

FIG. 5. Frictional forces associated with the different types of sliding modes of Fig. 5. "Pure liquid" sliding occurs with surfaces farther apart than 5-10 σ. "Liquidlike" sliding occurs with configurations as in Figs. 5(d) and 5(e), while "solidlike" sliding is associated with Figs. 5(a), 5(c), and 5(f). With certain liquids the sliding starts by being liquidlike and becomes progressively more solidlike during sliding; this is generally accompanied by a decrease in the film thickness and a "stress overshoot." An example of this given in the inset which shows measured data during an experiment with tetradecane. Note that a single stick-slip occurs over many microns and should not be confused with atomic scale stick-slip (Ref. 38) which may also be occurring but is beyond our resolution.
4. Viscosity and Newtonian fluids

**VISCOSITY**

\[ \tau = \text{shear stress (force/area)} \]

\[ \tau = \mu \frac{U}{H} \]

**Table 1: Viscosity of Common Liquids**

<table>
<thead>
<tr>
<th>liquid</th>
<th>temperature °C</th>
<th>( \mu ) gm/(cm.sec) = Poise</th>
<th>( \nu = \mu/\rho ) cm²/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>10</td>
<td>0.0131</td>
<td>0.0131</td>
</tr>
<tr>
<td>water</td>
<td>20</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>water</td>
<td>50</td>
<td>0.0055</td>
<td>0.0056</td>
</tr>
<tr>
<td>water</td>
<td>90</td>
<td>0.0031</td>
<td>0.0032</td>
</tr>
<tr>
<td>glycerine</td>
<td>20</td>
<td>17.6</td>
<td>14.0</td>
</tr>
<tr>
<td>mercury</td>
<td>0</td>
<td>0.014</td>
<td>0.001</td>
</tr>
<tr>
<td>lubricating oil</td>
<td>20</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>lubricating oil</td>
<td>40</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>lubricating oil</td>
<td>60</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** At 20°C, gases have a typical viscosity \( \mu \approx 10^{-4} \) gm/(cm.sec), but have a kinematic viscosity \( \nu = \mu/\rho \approx 0.1 \) cm²/sec.
5. On to the equations of motion

(a) stress versus rate-of-strain

(b) Navier-Stokes equations:
Assume that the material properties $\rho$ and $\mu$ are constant (generally an excellent approximation).

(c) pressure changes accompanying flow
- inertiially dominated: $\Delta p = O(\rho U^2)$
- viscously dominated: $\Delta p = O(\mu U / \ell)$
(d) incompressibility \( \nabla \cdot \mathbf{u} = 0 \)

variation of density accompanying motion should be small \( \Delta \rho \ll \rho \)

\[
\Delta \rho \approx \frac{\partial \rho}{\partial p} \Delta p, \quad c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s
\]

- inertially dominated flows: \( \frac{U}{c} \ll 1 \)
- viscously dominated flows: \( \left(\frac{U}{c}\right)^2 \ll \frac{\rho U \ell}{\mu} \)

(e) Reynolds number

\[
\text{REYNOLDS NUMBER:} \quad \mathcal{R} = \frac{\rho U \ell}{\mu} = \frac{U \ell}{\nu}
\]

Low-Reynolds-number motions: lubrication, film coating, suspensions, MEMS, ...

\[ \Rightarrow \quad 0 = -\nabla p + \mu \nabla^2 \mathbf{u} \]

(f) additional body forces:

- magnetic: ferrofluids

R.E. Rosensweig 1982 Magnetic Fluids. *Scientific American*


- electric: electric fields and dielectric materials, electrokinetic flows, electrophoresis
Elementary Ideas VIII

The Reynolds number

- Newton’s second law:

\[ \text{mass. acceleration} = \mathbf{\dot{a} \ forces} \]

High Reynolds number flow

Low Reynolds number flow

Forces (pressure) acting on fluid to cause motion

Friction from surrounding fluid which resists motion: \( \text{viscosity} \) \( (\mu) \)

\( \text{ratio of inertial effects to viscous effects in the flow} \)
5. Interfacial tension

(a) statics

capillary length: \[ \ell_{\text{cap}} = \left( \frac{\gamma}{\rho g} \right)^{\frac{1}{2}} \]

contact angle \( \Theta_c \)

\[ \gamma_{12} \cos \Theta_c = \gamma_{13} - \gamma_{23} \]

(b) dynamics

- drop deformation, formation of emulsions

(c) interfacial tension gradients: Maragoni motions

\[ \gamma(T), \quad \gamma(c) \]

Remark: tangential gradients of \( c \) or \( T \) give rise to tangential stresses that produce motion.
Quiz 1

• Consider the rise height of a liquid on a plane.

• Use dimensional arguments to show that the rise height is proportional to the capillary length.
PART II: Prototypical Flows I

Steady pressure-driven flow

**CHANNEL & PIPE FLOWS**

\[ u(y) = \frac{H^2 \Delta P}{2 \mu L} \left( 1 - \left(\frac{y}{H}\right)^2 \right) \]

\[ u(r) = \frac{R^2}{4 \mu} \frac{\Delta P}{L} \left( 1 - \left(\frac{r}{R}\right)^2 \right) \]

**NO-SLIP ON BOUNDARIES**

**AVG VELOCITY**

\[ \langle U \rangle = \frac{R^2}{8 \mu} \frac{\Delta P}{L} \]

**MASS FLOW RATE**

\[ Q = \frac{\pi}{8} \frac{R^4}{\mu} \frac{\Delta P}{L} \]

**PARABOLIC (POISEUILLE) VELOCITY PROFILE**

**APPLICATIONS:**

- Blood flow
- Pipe flow
- MEMS

**FILM FLOWS:**

- Liquid
- Inclined film
Additional effects when the mean free path of the fluid is comparable to the geometric dimensions

**GAS FLOW IN A MICROCHANNEL: COMPRESSIBLE FLOW WITH SLIP**

![Graph](image)

Figure 10: Helium mass flow for 1.33 micron channel, compared with Equation 23. The solid curve is the solution to Equation 23, assuming full tangential momentum accommodation and the dashed curve is the solution to Equation 23 setting $K = 0$ (no-slip solution).

**REF:** ARKILIC, SCHMIDT & BREUER
Prototypical Flows III

Even simple flows suffer dynamical instabilities!

Under "typical conditions", pipe flow becomes unstable when $R = \frac{U \cdot D}{\nu} > 2000$.

If (great) care is taken, the flow can be stable at (much) larger Reynolds number ($> 50000$).

Ref. D. Acheson
**Oscillatory Flows**

- Fluid
- Viscous boundary layer

\[ U \cos \omega t \]

Boundary layer \( \delta \) outside of which there is almost no motion.

\[ \delta \approx \left( \frac{\nu}{\omega} \right)^{1/2} \]

- As \( \nu \uparrow \), the viscous coupling increases.
- As \( \omega \uparrow \), the bulk knows less and less about the boundary motion.
Lubrication Flows I

**Theme:** Fluid motions characterized by long, narrow geometries.

**Typical Configurations**

1. Squeeze flow
2. Slider block
3. Pressure-driven flow
4. Plug in a closed cylinder
5. Journal bearing

**Characteristic Features**

(i) Pressure drop **along** the flow direction
(ii) Largest velocity gradients **across** the flow
Lubrication Flows II

LUBRICATION FLOWS: PRESSURE DROP VS VELOCITY

\[ P \quad \frac{H_0}{L} \quad y \quad \frac{L}{x} \quad P + \Delta P \]

NAVIER-STOKES EQNS: INERTIA IS NEGLIGIBLE

\[ \rho \left( \frac{\partial u}{\partial t} + u \cdot \nabla u \right) = -\nabla p + \mu \nabla^2 u \quad \text{[NEGLECT BODY FORCES]} \]

NEARLY

ONE-DIMENSIONAL FLOW

\[ \frac{\partial P}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} \]

\[ \therefore \quad \frac{\Delta P}{L} = O \left( \frac{\mu U}{H_0^2} \right) \]

"long length scale" ~ "small length scale"

APPLICATIONS OF THIS IDEA EXPLAIN A
WIDE RANGE OF LUBRICATION PHENOMENA.
Lubrication Flows III

Example: Cylindrical plug sedimenting in a closed tube

\[ R \]

Order-of-magnitude for the fall speed:

- \( U \): Fall speed
- \( u \): Typical fluid velocity in the gap

Mass conservation:

\[ U \pi R^2 \approx u \pi R (R_e) \]

Lubrication (pressure drop):

\[ \Delta P = \frac{\mu U L}{(R_e)^2} \]

Force:

\[ mg = F = \Delta P \pi R^2 \]

\[ U \propto \epsilon^3 \]

[Shear stresses on the sides are negligible]
Quiz 2

• Consider pressure-driven flow in a rectangular channel of height $h$ and width $w$ with $h << w$.

• Find an approximate expression for the flow rate through the channel.

• If the permeability is the ratio of the $\mu u/(\nabla p/L)$, find the permeability of such a rectangular channel.
Lubrication Flows IV

FILM COATING: LANDAU-LEVICH-DERJAGUIN PROBLEM

REGION II: FAR AWAY FROM THE PLATE THE FLUID IS NEARLY STATIC.
DEFORMED STATIC INTERFACE CHARACTERIZED BY:

\[ l_{\text{cap}} \approx \left( \frac{\gamma}{\rho g} \right)^{1/2} \]
CAPILLARY LENGTH

REGION I: SLOWLY VARYING FLOW ('VISCOUS ENTRAINMENT' VS 'CAPILLARY SUCTION')

\[ \frac{\mu U}{H^2} \approx \frac{\Delta p}{l} \]
\[ \Delta p \approx \frac{\gamma H}{l^2} \]
SLOWLY VARYING INTERFACE

"OVERLAP": CURVATURES OF REGIONS I & II AGREE:

\[ H \approx \frac{(\mu U)}{(\rho g)^{1/2} \gamma^{1/6}} \]

DETERMINES \( l \)
Lubrication Flows V

Film coating driven by surface tension gradients

"Climbing film"

Leading edge observed to go unstable

Flow driven by surface shear stress $\tau$

$$\tau = \left( \frac{d\gamma}{dT} \right) \frac{dT}{dz}$$

Material property

Applied temperature gradient

Lubrication approximation:

$$\frac{\Delta P}{l} \approx \frac{\mu u}{h^2} \approx \frac{\tau}{h}$$

Dynamic region connected to a static meniscus

$$H \approx \frac{\tau^2}{\gamma^{1/2} (\rho g)^{3/2}}$$

Film thickness $\propto \left( \frac{dT}{dz} \right)^2$

[Fanton, Cazabat & Quéré, 1996]
Lubrication Flows VI

Time-dependent geometries

SQUEEZE FLOW BETWEEN TWO DISKS

TWO CIRCULAR DISKS ARE SQUEEZED TOGETHER WITH A CONSTANT FORCE.

SEPARATION DISTANCE VS TIME?

MASS CONSERVATION

\[ U \pi R^2 \approx u \cdot 2\pi RH \quad \rightarrow \quad u = \frac{UR}{H} \]

LUBRICATION

\[ \frac{\Delta P}{R} = \frac{\mu u}{H^2} \approx \frac{\mu UR}{H^3} \]

FORCE

\[ F \approx (\Delta P) \pi R^2 \approx \frac{\mu UR^4}{H^3} \]

GAP THICKNESS

\[ U = \frac{dH}{dt} \quad \rightarrow \quad \frac{dH}{dt} \propto H^3 \]

\[ \therefore H \approx t^{\frac{3}{2}} \quad \text{for large times} \]
Lubrication Flows VII

**SPREADING FILMS**

DYNAMICS OF LIQUIDS SPREADING ON SOLID (OR LIQUID) SUBSTRATES

**MODEL PROBLEMS**

- **Gravitationally Driven Spreading**
- **Surface-Tension (Capillary) Driven Spreading**
- **Coating of a Spinning Disk**
- **Surface Tension Gradients Drive Spreading**

SURFACANTS

**EACH SPREADING CONFIGURATION IS CHARACTERIZED BY A LONG, NARROW REGION OF FLOW**

LUBRICATION APPROXIMATION
Lubrication Flows VIII

SPREADING RATES

\[ h(x,t) \]
\[ L(t) \]

SOME IMPORTANT IDEAS

1. DETERMINE \( L(t) \), \( h(x,t) \)
   TYPICAL VELOCITIES \( u \approx \frac{dl}{dt} \approx \frac{L(t)}{t} \)

2. COMBINE MOMENTUM BALANCE
   WITH A GLOBAL MASS BALANCE

   2D \( h \cdot L \approx \text{constant} \)

   AXISYMMETRIC \( hL^2 \approx \text{constant} \)

3. DRIVING FORCES

   GRAVITY \( \Delta p \approx \rho g h \)

   CENTRIFUGAL \( \Delta p \approx \frac{1}{2} \sigma L^2 \cdot L^2 \)

   SURFACE TENSION \( \Delta p \approx \delta h/L^2 \)

   (NOT DRIVEN BY SPREADING COEFFICIENT)

   de Gennes 1985
Lubrication Flows IX

**SCALING LAWS**

The evolution of the film shape can be predicted by solving a nonlinear PDE.

\[
\frac{\partial h}{\partial t} = \frac{\rho g}{3\mu} \frac{\partial}{\partial x} \left( h^3 \frac{\partial h}{\partial x} \right) + \text{global mass conservation}
\]

Spreading of a 2D gravity current:

\[
L(t) \approx t^\alpha
\]

<table>
<thead>
<tr>
<th>DRIVING FORCE</th>
<th>SPREADING RATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity</td>
<td>( \alpha = \frac{1}{5} = \frac{1}{8} )</td>
</tr>
<tr>
<td>Rotation (spin coating)</td>
<td>( \alpha = \frac{1}{4} )</td>
</tr>
<tr>
<td>Surface tension</td>
<td>( \alpha = \frac{1}{7} = \frac{1}{10} ) (Tanner's law)</td>
</tr>
</tbody>
</table>

2D axisymmetric

3D axisymmetric
Suspension Flows I

**SEDIMENTATION**

- **DEFINITION OF VISCOSITY**

\[
\text{SHEAR STRESS} = \tau = \mu \frac{\partial u}{\partial y}
\]

- **SEDIMENTING SPHERE**

\[
F_{\text{HYDRO}} = F_{\text{external}} = \rho_p \frac{4\pi a^3}{3} g
\]

(ARCHIMEDES)

\[
-p \frac{4\pi a^3}{3} g = F_{\text{HYDRO}} - F_{\text{buoyant}}
\]

Order of magnitude

\[
F_{\text{HYDRO}} \propto \text{SHEAR STRESS} \cdot \text{AREA} \propto -\frac{\mu U}{a} \frac{4\pi a^2}{a} \propto -4\pi \mu a U
\]

STOKES:

\[
F_{\text{HYDRO}} = -6\pi \mu a U
\]

\[
U = \frac{2a^2(\rho_p - \rho)}{9\mu}
\]

\[\rightarrow \quad a^2 \text{ dependence}\]
Suspension Flows II

SEDIMENTATION: SLENDER PARTICLES

\[ \frac{a}{L} \ll 1 \]

\[ U_\parallel \approx 2U_\perp \]

Force = \frac{\text{force}}{\text{length}} \cdot \text{length}

\approx O(\mu U \cdot 2L)

(DIMENSIONAL ARGUMENT)

INDEPENDENT OF a?! \Rightarrow THE PROBLEM WITH A PURELY DIMENSIONAL ARGUMENT!

LOW REYNOLDS NUMBERS: \[ Re = \frac{Ua}{\nu} \]

(i) \[ 1 \ll \frac{L}{a} \ll Re^{-1} \]

\[ F \approx 2\pi \mu U L \frac{1}{\ln(L/a)} \]

(ii) \[ 1 \ll Re^{-1} \ll \frac{L}{a} \]

\[ F \approx 2\pi \mu U L \frac{1}{\ln(Re^{-1})} \]

APPLICATION TO THE PROPULSION OF SWIMMING MICROORGANISMS.

[ BERG & PURCELL]
Suspension Flows III

SEDIMENTATION VELOCITY OF SMALL DROPS

\[ F_{\text{HYDRO}} = -4\pi \alpha \mu U \left( \frac{1 + \frac{3}{2} \lambda}{1 + \lambda} \right) \]

\[ U \sim \left( \frac{\rho_d - \rho}{3 \mu} \right) a^2 g \left( \frac{1 + \lambda}{1 + \frac{3}{2} \lambda} \right) \]

\[ \frac{U_{\lambda=0}}{U_{\lambda=\infty}} = \frac{2}{3} \quad \text{only} \]

Clean Interfaces

NOTE: SURFACANTS CAN HAVE A SIGNIFICANT INFLUENCE. FREQUENTLY, SMALL DROPS RISE LIKE RIGID SPHERES.

SEDIMENTS ALMOST LIKE A RIGID SPHERE

SURFACANTS PRODUCE SURFACE TENSION GRADIENTS WHICH PRODUCE A NEARLY RIGID INTERFACE.
**Effective Viscosity of a Suspension**

- **Dilute Suspension** $\phi \ll 1$

  - On average expect a resistance as if the medium were homogeneous with an effective viscosity

    $$\mu_{\text{eff}} = \mu \left[ 1 + \frac{5}{2} \phi \right]$$  \textit{(Einstein 1906)}

- If the suspended particles are droplets (spherical) of viscosity $\lambda \mu$, then

  $$\mu_{\text{eff}} = \mu \left[ 1 + \phi \left( \frac{1 + \frac{5\lambda}{2}}{1 + \lambda} \right) \right]$$  \textit{(Taylor 1932)}

- Slender rods:

  "Effective Viscosity" $\propto \phi \left( \frac{L}{a} \right)^2$

  \textit{(Batchelor 1990)}
Suspension Flows IV

Brownian motion and diffusion:
The Stokes-Einstein equation

- A typical diffusive displacement in time $\tau$ are linked by
  \[(\text{distance})^2 = D_t \tau.\]

- Translation diffusion of spherical particles
- Einstein: related thermal fluctuations to mean square displacement; with resistivity: $\zeta = \text{force/velocity}$
  \[\text{where } \zeta = F/U\]

- Stokes: $\zeta = 6\pi \mu a$

- Typical magnitudes (small molecules in water):
  \[\text{... can also investigate other shapes, rotational diffusion}\]
Suspension Flows VI

The Physical System

Supported Membranes [Sackmann 1996, Science]

The Hydrodynamic Model
(Saffman-Delbrück, 1975,6)

Translational Diffusion Coefficient

\[ D_T = \frac{k_B T}{(F/U)} \]
PARTICLE MOTION IN MEMBRANES

PREDICTED TRANSLATIONAL DIFFUSION COEFFICIENTS FOLLOW FROM

\[ D = \frac{k_B T}{(F/U)} \]

HYDRODYNAMIC MODEL ACCOUNTING FOR SURROUNDING FLUID AND A NEARBY RIGID BOUNDARY SUGGEST

(\text{SAFFMAN} 1976) \[ F = -\frac{4\pi \mu RU}{\Lambda \left[ \ln \left( \frac{L}{\Lambda} \right) - \gamma \right]} \quad (H \to \infty) \quad \gamma = 0.57 \]

(\text{STONE \& AJDARI} 1998) \[ F = -\frac{4\pi \mu RU}{\Lambda \left[ \ln \left( \frac{\Lambda R}{4H} \right)^{1/2} - \gamma \right]} \quad (\text{FINITE } R/H) \]

\[ \Lambda = \frac{\mu R}{\mu m h} \]

\text{MATERIAL PARAMETER CHARACTERIZING VISCOUS RESISTANCE}

[SEE ALSO EVANS \& SAAFMMAN (1989)]

Ref. Stone \& Ajdari 1996
MARANGONI Flows: Surface-driven motions

THERMOPHORESIS

Small bubbles (or drops) in a liquid are observed to translate in a temperature gradient. Surface tension $\gamma(T)$

\[ \gamma = \frac{2a (d\gamma/dT)}{(2+\delta)(2+3\lambda)\mu} \gamma T \]

Typically, $\frac{d\gamma}{dT} < 0$

$\Rightarrow$ may dominate buoyancy-driven motions for $a < 10 \mu m$.

[Young, Block & Goldstein 1959]

Note: an 'explanation' based upon Brownian motion and thermally-enhanced collisions would predict that the drop translates in the direction hot $\rightarrow$ cold.
More on thermally-driven flows

THERMALLY DRIVEN LUBRICATION FLOW WHICH MAINTAINS SEPARATION OF THE INTERFACES

REFERENCE: DELL’AVERSA & NEISTEL
PHYSICS TODAY, JANUARY 1998
Gradients in surface tension: Marangoni stresses

- Local value of surface tension is altered by change of temperature or surfactant concentration

\[ \text{air} \quad \text{low tension} \quad \text{high tension} \quad \text{liquid} \quad \text{Fluid dragged from low to high tension} \]

- Contaminants typically lower surface tension
- Example: alcohol and water

surfactants: amphiphilic molecules

Hydrocarbon tail \[ \text{air} \quad \text{water} \]

Polar head group

Courtesy of Professor Maria Teresa Aristodemo, Florence, and Dr. Raffaele Savino, Naples

Carlo Marangoni (1840–1925)
Gradients in surface tension: Marangoni stresses

- Local value of surface tension is altered by change of temperature or surfactant concentration

- Contaminants typically lower surface tension
- Example: alcohol and water

surfactants: amphiphilic molecules (soap)

Fluid dragged from low to high tension

Courteous of Professor Maria Teresa Aristodemo, Florence, and Dr. Raffaele Savino, Naples

Carlo Marangoni (1840–1925)
Wine tears

An example of the Marangoni effect

Evaporation from thin film
Wine tears

An example of the Marangoni effect

Evaporation from thin film

high surface tension

Marangoni stress

low surface tension
In fact: an improved history

• Fluid motions due to gradients in surface were first properly described by James Thomson in 1855

  On certain curious motions observable at the surfaces of wine and other alcoholic liquors

• James Thomson was the older brother of William Thomson (who will appear later in the talk)
Conclusions

• Continuum descriptions of fluid-like systems begin with momentum statement involving stress (Cauchy equation)

• For Newtonian fluids the starting point is the Navier-Stokes equations which is commonly studied assuming the density and viscosity are constant

• Common geometric configurations, including thin films, are well studied and ammenable to analysis

• Many common features among areas of complex fluids, suspensions, lubricating films, etc.
TIME-DEPENDENT EVOLUTION OF FLOWS

1. Fluid responds on a "diffusive" time scale when a nearby boundary is moved.

   \[ u(y,t) = U_f \left( \frac{y}{\sqrt{vt}} \right) \]

   Fluid initially at rest

   Diffusion time \( \propto \frac{l^2}{v} \)
   Diffusion distance \( \propto \sqrt{vt} \)

2. Entrance length: Distance along a pipe to establish a parabolic velocity profile

   \[ R \sim \left( \frac{v \cdot \text{time}}{L} \right)^{1/4} \]

   \[ \frac{L}{R} \propto \frac{RU}{v} \]

   Experiment: \( L \approx 0.1(Re)R \)
HIGH REYNOLDS NUMBER FLOWS

1. CHANGE IN PRESSURE ACCOMPANYING HIGH SPEED FLOW \[ R = \frac{U}{\nu} \gg 1 \]
   \[ \Delta P \propto \rho U^2 \]
   FORCE ON OBJECT \( \propto \rho U^2 l^2 \) (flow separation)
   \( \Rightarrow \) LIFT FOR FLYING: SEE THE "GREAT FLIGHT DIAGRAM"

2. VISCOUS BOUNDARY LAYER:

   \[ \frac{\rho U^2}{x} \simeq \frac{\mu U}{\delta^2} \]

   boundary layer thickness

   \[ \delta(x) \propto \left( \frac{1}{x} \right)^{1/2} \] [AGAIN \( \frac{1}{2} \)]
Figure 2 The Great Flight Diagram. The scale for cruising speed (horizontal axis) is based on equation 2. The vertical line represents 10 meters per second (22 miles per hour).

REFERENCE: H. Tennekes "THE SIMPLE SCIENCE OF FLIGHT"
FIG. 6. A sequence of pictures of a water drop falling from a circular plate 1.25 cm in diameter (Shi, Brenner, and Nagel, 1994). The total time elapsed during the whole sequence is about 0.1 s. Reprinted with permission. © American Association for the Advancement of Science.

FIG. 7. A drop of a glycerol and water mixture, 100 times as viscous as water, falling from a nozzle 1.5 mm in diameter. As opposed to the case of water, a long neck is produced (Shi, Brenner, and Nagel, 1994). Reprinted with permission. © American Association for the Advancement of Science.