APPLIED PHYSICS 298R
Stress, Strain and All That Jazz

John W. Hutchinson
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OUTLINE OF LECTURE 5

Continuum Mechanics in Thirty Minutes:

Stresses – normal and shear
Strains – stretching and shear
Linear elastic solids – Young' s modulus, Poisson's ratio
General elastic moduli
Linear viscous fluid – viscosity
Linear visco-elastic solids
Strength of solids

References:

Engineering Materials, An Introduction, M. G. Ashby and D. R. H. Jones, Pergamon Press, 1980.

Foundations of Solid Mechanics, Y. C. Fung, Prentiss-Hall, 1965.

LINEAR ELASTIC BEHAVIOR, STRESS + STRAIN

$\sigma \sim$ normal stress $\left(\frac{Nt}{m^2}, Pa\right)$

$\varepsilon \sim$ stretching strain

$= \dfrac{\text{change in length}}{\text{original length}}$

YOUNG'S MODULUS (Pa)

UNIAXIAL BEHAVIOR

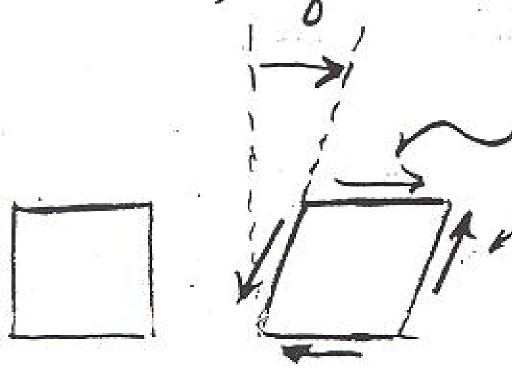
$\sigma = E\,\varepsilon$

$(\varepsilon_T = -\nu\,\varepsilon)$

SHEAR STRAIN $\rightarrow \gamma$

POISSON'S RATIO $-1 < \nu < \frac{1}{2}$

$\tau \sim$ SHEAR STRESS (Pa)



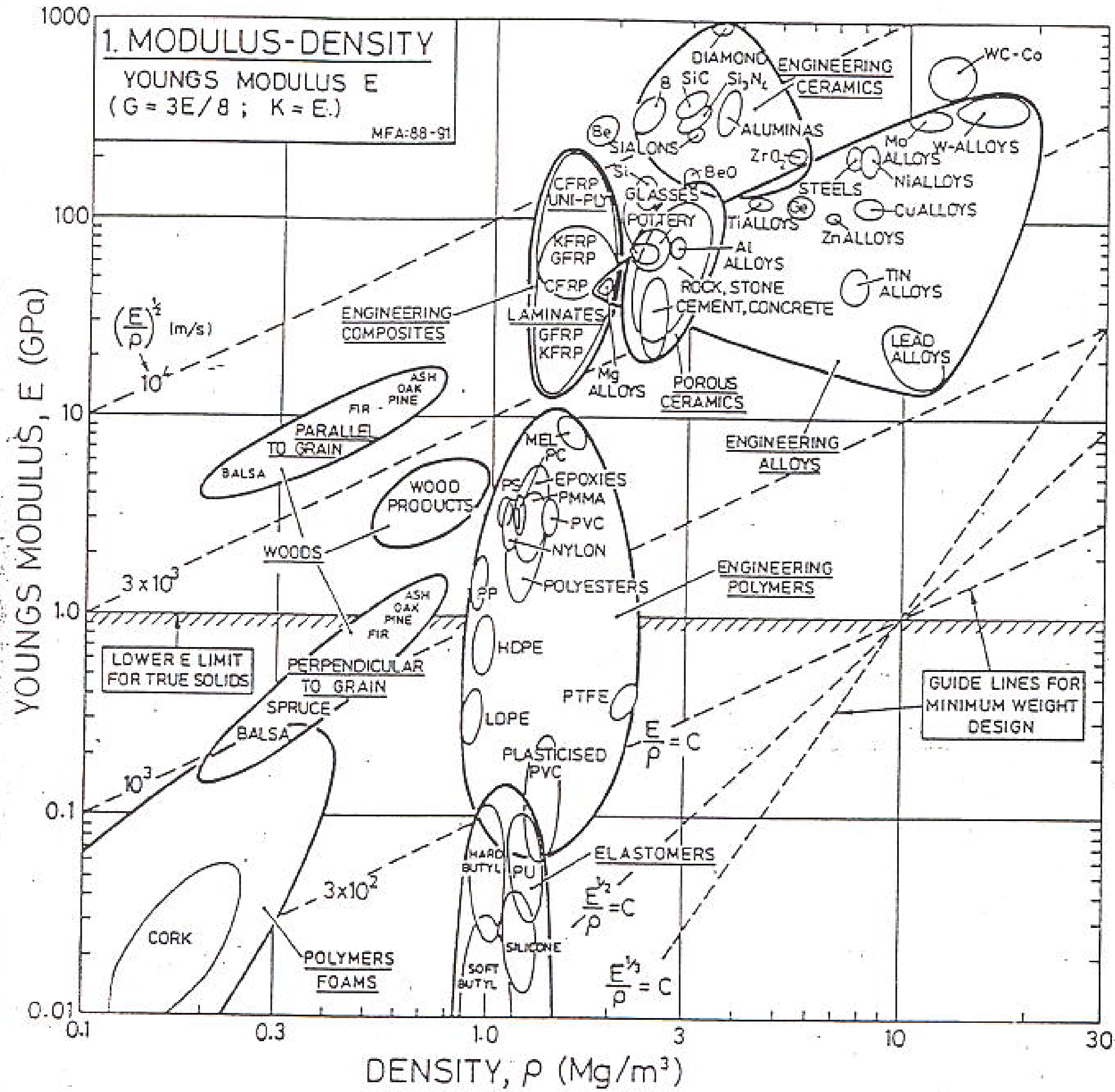
$\tau = \mu\,\gamma$

SHEAR MODULUS (Pa)

ISOTROPIC SOLID

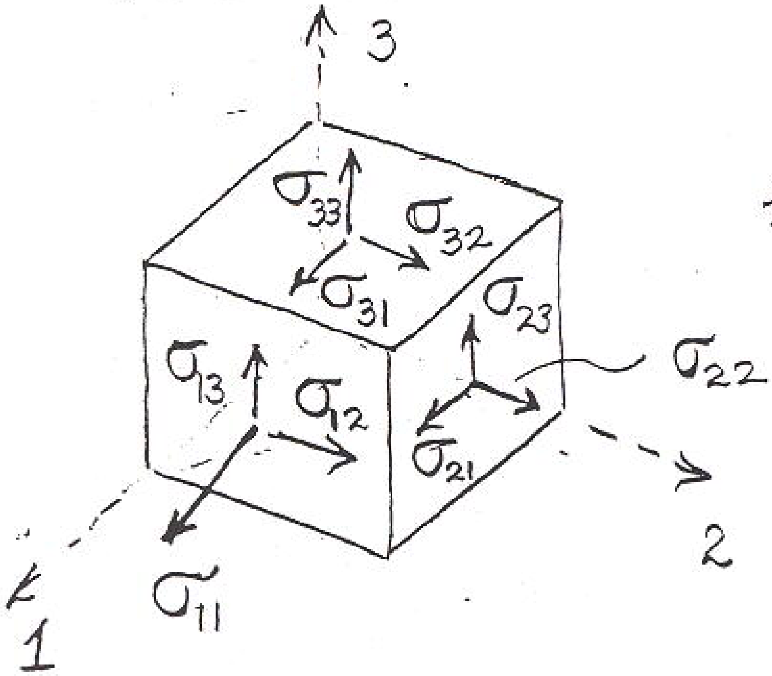
2 INDEPENDENT ELASTIC CONSTANTS

$$\mu = \frac{E}{2(1+\nu)} \qquad k = \frac{E}{3(1-2\nu)} \quad \text{(BULK MODULUS)}$$



From "MATERIALS SELECTION IN MECHANICAL DESIGN", M. F. Ashby, Pergamon 1992

3-D STRESS + STRAIN



σ_{ij} ← direction of stress

↗ face

$$(\sigma_{ij} = \sigma_{ji})$$

$$\varepsilon_{ij} \quad \begin{cases} \varepsilon_{11} \\ \varepsilon_{12} = \varepsilon_{21} = \dfrac{\gamma_{12}}{2} \end{cases}$$

$$(\varepsilon_{ij} = \varepsilon_{ji})$$

LINEAR ELASTIC BEHAVIOR

$$\sigma_{ij} = E_{ijkl}\,\varepsilon_{kl}$$

(In general, 21 independent elastic constants)

ISOTROPIC SOLID (See Page 5)

$$\sigma_{ij} = \frac{E}{1+\nu}\left[\varepsilon_{ij} + \frac{1}{1-2\nu}\varepsilon_{kk}\,\delta_{ij}\right]$$

STRESS-STRAIN RELATION

Isotropic Linear Elastic Solid

$$\varepsilon_{11} = \frac{\sigma_{11}}{E} - \frac{\nu}{E}\left(\sigma_{22} + \sigma_{33}\right)$$

$$\varepsilon_{22} = \frac{\sigma_{22}}{E} - \frac{\nu}{E}\left(\sigma_{11} + \sigma_{33}\right)$$

$$\varepsilon_{33} = \frac{\sigma_{33}}{E} - \frac{\nu}{E}\left(\sigma_{11} + \sigma_{22}\right)$$

$$\varepsilon_{12} = \frac{1}{2\mu}\sigma_{12}\ , \quad \varepsilon_{13} = \frac{1}{2\mu}\sigma_{13}\ , \quad \varepsilon_{23} = \frac{1}{2\mu}\sigma_{23}$$

Tensor Notation:

$\sigma_{11} + \sigma_{22} + \sigma_{33}$

$$\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\,\delta_{ij} \qquad \mu = \frac{E}{2(1+\nu)}$$

mean stress

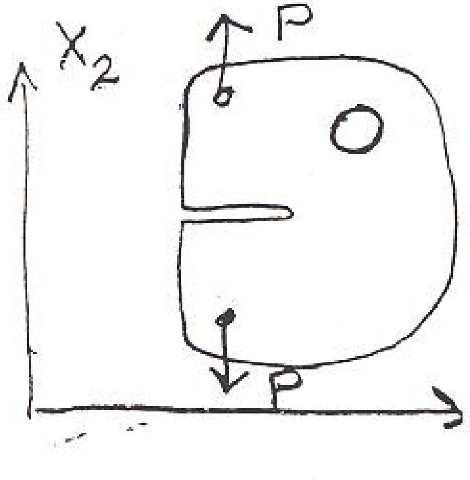
$$\varepsilon_{jj} = \frac{1}{3K}\sigma_{jj} \qquad K = \frac{E}{3(1-2\nu)}$$

bulk modulus

dilatation

ELASTICITY THEORY:

EQUATIONS GOVERNING DEFORMATION OF A LINEAR ELASTIC SOLID (2-D)



$\sigma_{ij}(x_1, x_2)$ $\quad i = 1, 2$

$\varepsilon_{ij}(x_1, x_2)$ $\quad j = 1, 2$

+time

$u_1(x_1, x_2)$, $u_2(x_1, x_2)$

displacements

STRAIN-DISPLACEMENT

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1}, \quad \varepsilon_{22} = \frac{\partial u_2}{\partial x_2}$$

$$\varepsilon_{12} = \varepsilon_{21} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$$

EQUILIBRIUM

density

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} = \rho \frac{\partial^2 u_1}{\partial t^2}$$ acceleration

$$\frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} = \rho \frac{\partial^2 u_2}{\partial t^2}$$

STRESS-STRAIN (ISOTROPIC)

$$\varepsilon_{11} = \frac{1}{E}\left(\sigma_{11} - \nu \sigma_{22}\right), \quad \varepsilon_{22} = \frac{1}{E}\left(\sigma_{22} - \nu \sigma_{11}\right)$$

$$\varepsilon_{12} = \frac{1+\nu}{E}\,\sigma_{12}$$

LINEAR VISCOUS FLUIDS

(Newtonian Fluid)

$$\dot{\varepsilon}_{kk} = \frac{1}{3K}\dot{\sigma}_{kk} \qquad \dot{\gamma} = \frac{1}{\beta}\tau$$

time rate

viscosity

$\Rightarrow \dot{\varepsilon}_{kk} \cong 0$

(incompressible)

TENSOR FORM (Isotropic)

$$\dot{\varepsilon}_{kk} = 0 \quad , \quad \dot{\varepsilon}_{ij} = \frac{1}{\beta} S_{ij}$$

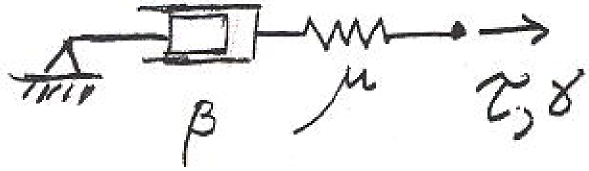
deviator stress: $S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$

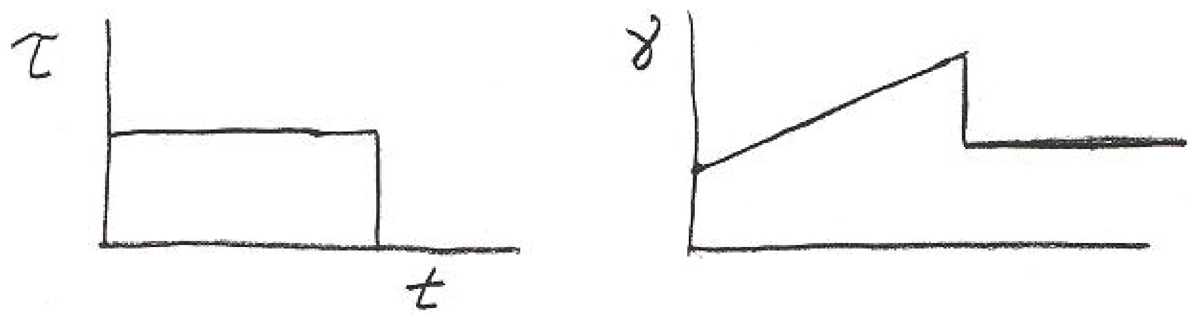
β is a strong function of temperature, additives, etc.

ANY NON-HYDROSTATIC STRESS PRODUCES A STRAIN RATE, i.e. FLOW

LINEAR VISCO-ELASTIC SOLIDS

$$\dot{\gamma} = \underbrace{\frac{1}{\mu}\dot{\tau}}_{\text{elastic}} + \underbrace{\frac{1}{\beta}\tau}_{\text{viscous}}$$



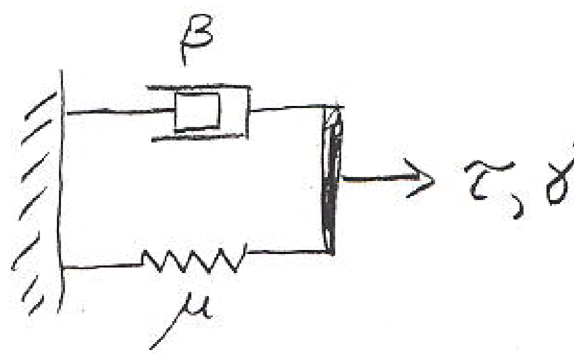


time constant: $t_0 = \frac{\beta}{\mu}$

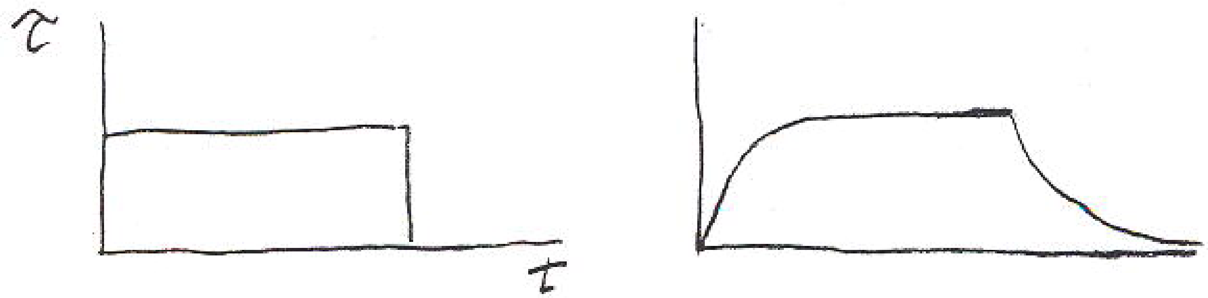
If $t << t_0$: elastic behavior

$t >> t_0$: viscous behavior

Alternative Model



$$\tau = \mu\gamma + \beta\dot{\gamma}$$

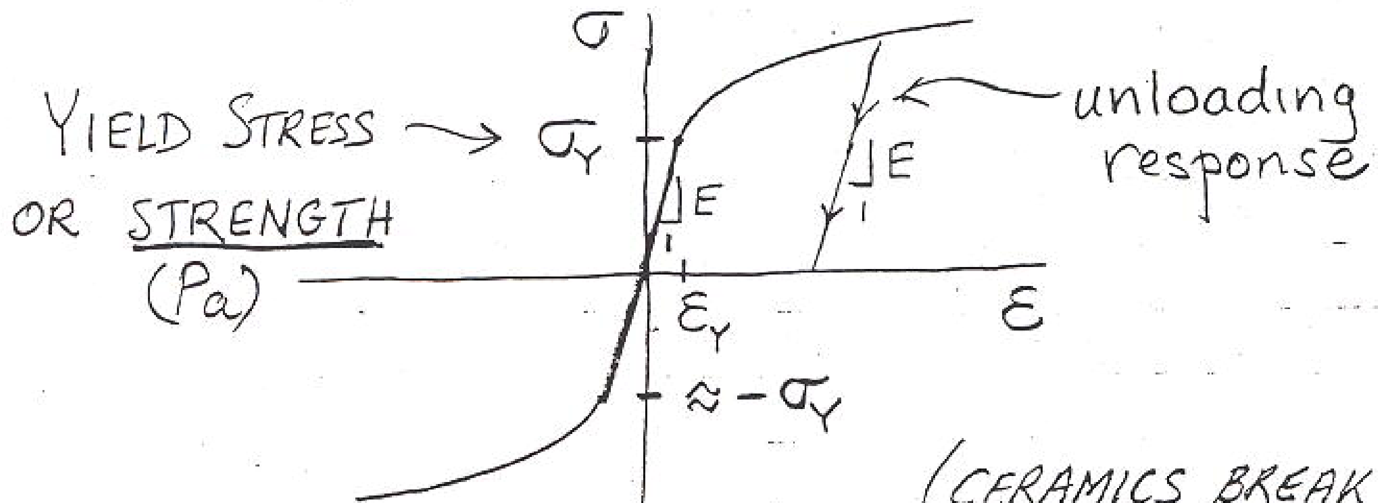


NONLINEAR BEHAVIOR (PLASTICITY)

HIGH TEMPERATURES $\Longrightarrow$ TIME-DEPENDENT BEHAVIOR, CREEP, ...

$T < \frac{1}{2} T_{melt}$ $\Longrightarrow$ BEHAVIOR OF METALS + CERAMICS $\approx$ INDEPENDENT OF TIME

<u>UNIAXIAL BEHAVIOR</u> OF METALS & POLYMERS IN TIME-INDEPENDENT REGIME



(CERAMICS BREAK IN TENSION!)

$\varepsilon_Y \equiv \frac{\sigma_Y}{E}$ FOR MOST METALS + POLYMERS $10^{-3} < \varepsilon_Y < 10^{-2}$

INDENTATION TEST FOR <u>HARDNESS</u>

$H = \frac{P}{A} \approx 3\sigma_Y$

(Pa)

STRENGTH σ_s ← Pa

BRITTLE MATERIALS IN TENSION:
CERAMICS, GLASSES, ...

σ_s ~ catastrophic fracture due to small flaws

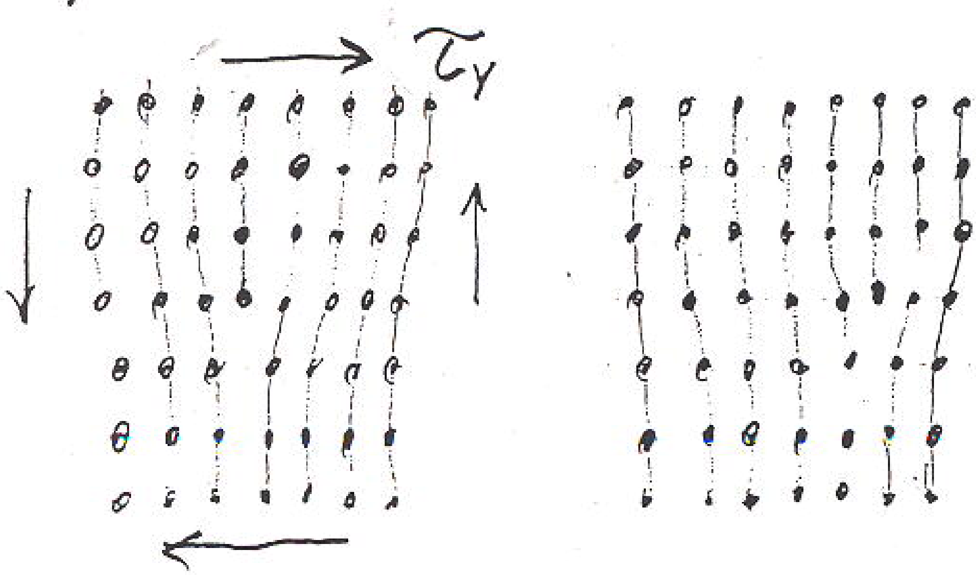
IN COMPRESSION A VARIETY OF MECHANISMS, INCLUDING PLASTIC FLOW

$$(\sigma_s)_{compression} \gg (\sigma_s)_{Tension}$$

DUCTILE METALS (TENSION OR COMPRESSION)

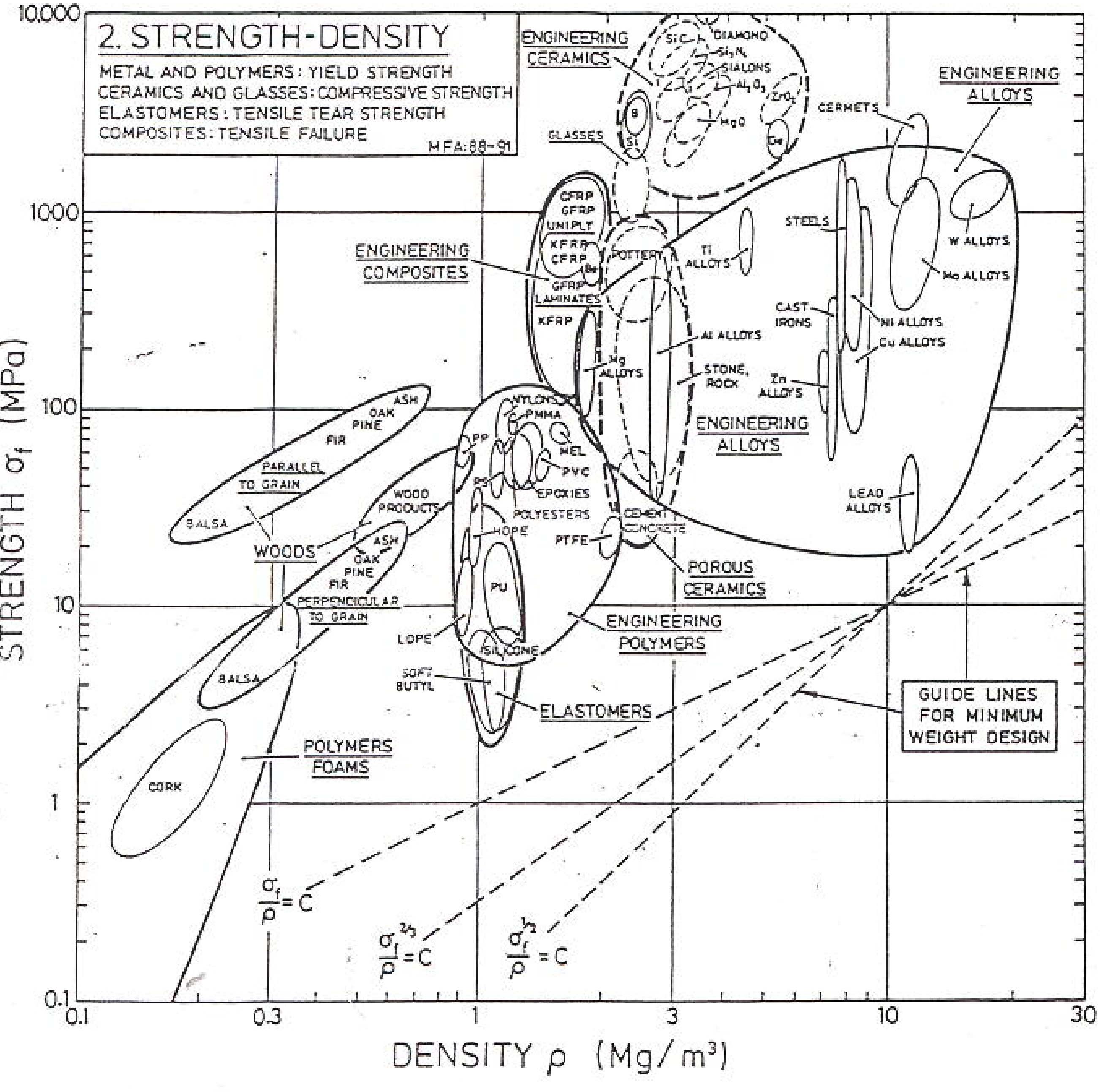
$\sigma_s \equiv \sigma_Y$ ~ stress for plastic yielding

plastic flow due to dislocation motion



$$\tau_Y \ll \mu/10$$

11,



From M.F. Ashby, 1992